LRFD

Section 1.3

Revised: June 2006

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1.1 Properties for Column and Pile Bents

The moment of inertia shall be computed for any types, sizes, and number of piles to be used that are not given in these tables.

Concrete Columns

f'_c = 3 ksi E_c = 3,156 ksi
f'_c = 4 ksi E_c = 3,644 ksi

Table 1.3.1.1.1 Gross Moment of Inertia of Concrete Columns

		Number of Columns						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Column Diameter, ft.	**2.5**	39,760	79,520	119,280	159,040	198,800	238,560	278,320
	3	82,448	164,896	247,344	329,792	412,240	494,688	577,136
	3.5	152,745	305,490	458,235	610,980	763,725	916,470	1,069,215
	4	260,576	521,152	781,728	1,042,304	1,302,880	1,563,456	1,824,032
	4.5	417,393	834,786	1,256,179	1,669,572	2,086,965	2,504,358	2,921,751

Steel Pile

E_s = 29,000 ksi

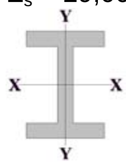

Table 1.3.1.1.2 Gross Moment of Inertia of Steel Piles

	I_{xx}, in.^{4}			I_{yy}, in.^{4}		
	Pile Size			Pile Size		
# of Piles	HP 10 x 42	HP 12 x 53	HP 14 x 73	HP 10 x 42	HP 12 x 53	HP 14 x 73
1	210	393	729	71.7	127	261
“n”	210 x “n”	393 x “n”	729 x “n”	71.7 x “n”	127 x “n”	261 x “n”

Table 1.3.1.1.3 Alternate Pile Properties *

Gross Moment of Inertia, in.^{4}	**f'_c = 4 ksi**
14 in. ϕ C.I.P. Pile, I = 1886	E’ = 8657 ksi *
20 in. ϕ C.I.P. Pile, I = 7854	E’ = 7239 ksi *
24 in. ϕ C.I.P. Pile, I = 16,286	E’ = 6668 ksi *

* To account for the composite material properties as well as the geometric properties of the C.I.P. pile, apply the equation, $E'I = E_sI_s + E_cI_c$. Where E' is the equivalent modules of elasticity associated with the total moment of inertia, I. This will allow the longitudinal force distribution program to compute the correct stiffness for the bent containing the C.I.P. piles. Steel pipe properties are calculated assuming the following:

- Outside Diameter = C.I.P pile diameter
- 3/8” Design Thickness = (1/2” nominal thickness) – (12.5% fabrication tolerance) – (1/16” deterioration)

1.2 Longitudinal Bent Stiffness/Resultant Moment of Inertia

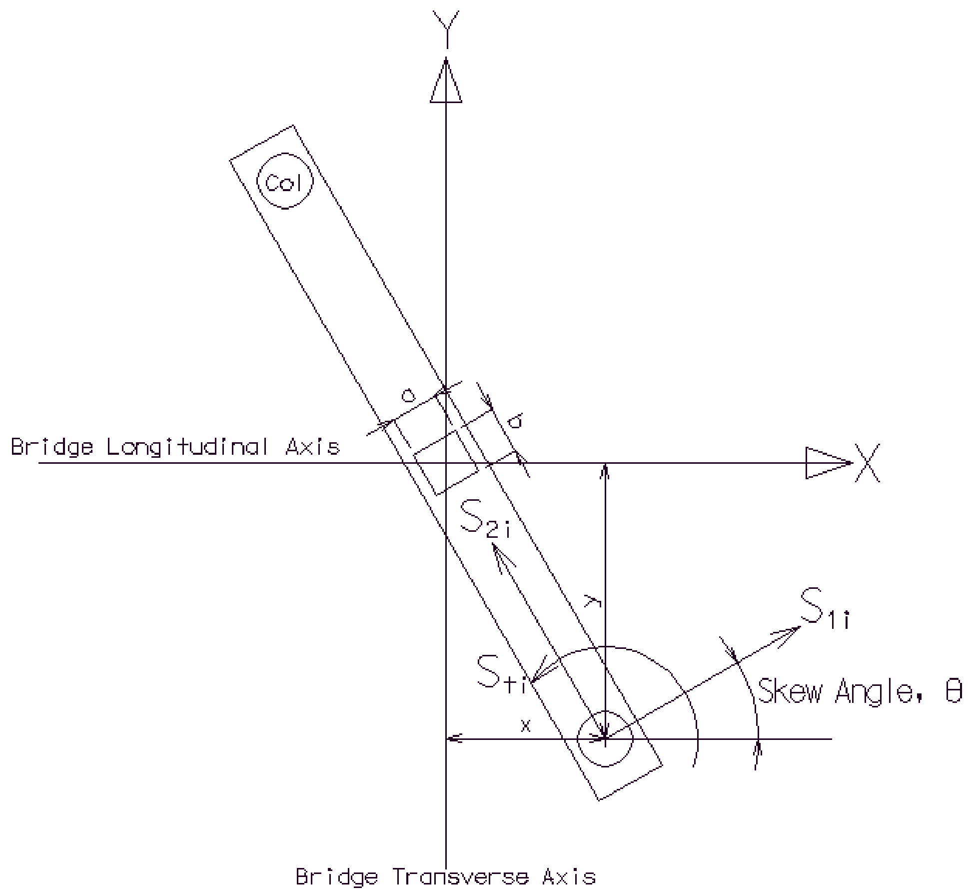

Figure 1.3.1.2.1 Longitudinal Bent Stiffness Diagram

Where:
S_{1i} = Stiffness of the i^{th} column normal to the bent (units - force/length)
S_{2i} = Stiffness of the i^{th} column parallel to the bent
S_{ti} = Torsional stiffness of the i^{th} column
θ = Skew angle (positive in counterclockwise direction)
X_i = Coordinate distance from the bent origin to the i^{th} column considered long the bridge longitudinal axis (+/-)
Y_i = Coordinate distance from the bent origin to the i^{th} column considered along the bridge transverse axis (+/-)
e_i = $-Y_i \cos(\theta) + X_i \sin(\theta)$
f_i = $X_i \cos(\theta) + Y_i \sin(\theta)$
N = total number of columns

Moment of Inertia for a Skewed Bent

In the distribution of loads in the bridge longitudinal direction, the stiffness in the bridge longitudinal and transverse directions is coupled for a skewed bent. Therefore, the bent will experience a deflection in the bridge longitudinal direction and the bridge transverse direction simultaneously. To account for this coupling effect, Matrix Structural Analysis is used here to determine the bent stiffness matrix which consists of stiffnesses S_1, S_2, and S_t of all individual columns.

To simplify this analysis, use the following procedure.

Moment of inertias of an individual column - Round

$$I_y = \frac{\pi \times r^4}{4}, \; I_z = \frac{\pi \times r^4}{4}, \; J = \frac{\pi \times r^4}{2}$$

Moment of inertias of an individual column - Rectangular

$$I_y = \frac{b \times a^3}{12}, \; I_z = \frac{a \times b^3}{12}, \; J = \frac{(a \times b)(a^2 + b^2)}{12}$$

Where:

I_y = Column moment of inertia parallel to the bent (in.^{4})
I_z = Column moment of inertia perpendicular to the bent (in.^{4})
J = Polar moment of inertia (in.^{4})
r = Radius of a circular column (in.)
a = Width of column normal to the bent (in.)
b = Width of column parallel to the bent (in.)

Stiffness of the individual column

After calculating the inertias of the columns the stiffness of the bent can be figured from the following.

$$S_1 = \frac{3 \times E \times I_y}{{L_1}^3}, \; S_2 = \frac{12 \times E \times I_z}{{L_2}^3}, \; S_t = \frac{G \times J}{L_3}$$

Where:

E = Modulus of elasticity of the column (ksi)
S_1 = Stiffness of the individual column normal to the bent. (kip/in.)
L_1 = Unsupported length from the top of the beam to the bottom of the column. (in)
S_2 = Stiffness of the individual column parallel to the bent. (kip/in.)
L_2 = Unsupported length from the bottom of the beam to the bottom of the column. (in.)
S_t = Torsional stiffness of the individual column. (kip-in./rad)
L_3 = Average of the two lengths calculated for S_1 and S_2. (in)
G = Shear modulus of the column. (ksi)

The stiffness difference in each direction comes from the column-beam interaction. In the direction normal to the bent, the column is considered fixed at the bottom and allowed to freely deflect and rotate at the top. In the direction parallel to the bent however, the top. Notice also that the unsupported lengths are different in each direction. (See Figure) The above equations may then be derived using the slope deflection method.

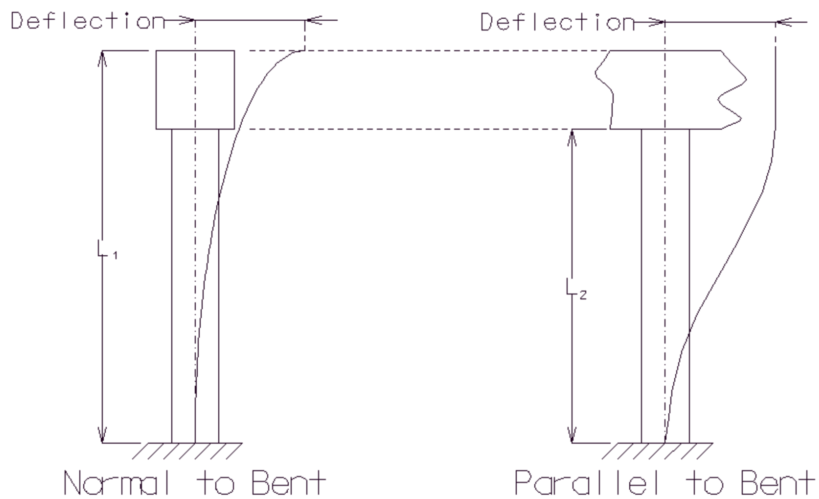

Figure 1.3.1.2.2 Unsupported Lengths for Stiffness Calculations

Stiffness Coefficients of Bent

$$C_1 = \sum_{i=1}^{N}\left(\cos^2(\theta)\times S_{1i} + \sin^2(\theta)\times S_{2i}\right)$$

$$C_2 = \sum_{i=1}^{N}\left(e_i \times \cos(\theta)\times S_{1i} - f_i \times \sin(\theta)\times S_{2i}\right)$$

$$C_3 = \sum_{i=1}^{N}\left(\cos(\theta)\times \sin(\theta)\times \left(S_{1i} - S_{2i}\right)\right)$$

$$C_4 = \sum_{i=1}^{N}\left(S_{ti} + \left(e_i^{\,2} \times S_{1i}\right) + \left(f_i^{\,2} \times S_{2i}\right)\right)$$

$$C_5 = \sum_{i=1}^{N}\left(e_i \times \sin(\theta)\times S_{1i} + f_i \times \cos(\theta)\times S_{2i}\right)$$

$$C_6 = \sum_{i=1}^{N}\left(\sin^2(\theta)\times S_{1i} + \cos^2(\theta)\times S_{2i}\right)$$

Resultant Longitudinal Stiffness

$$S_r = A_3 - \frac{A_2}{A_1} \text{ (k/in.)}$$

Where:

A_1 = $(C_4)(C_6) – (C_5)^2$
A_2 = $(C_2)^2(C_6) - 2(C_2)(C_3)(C_5) + (C_3)^2(C_4)$
A_3 = C_1

Resultant Moment of Inertia

Thus the resultant moment of inertia for the bent about the bridge longitudinal axis can be expressed as

$$I_r = \frac{L_3^{\,3}}{3\times E}\left[S_r\right]$$